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Code No: 153SB

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech II Year I Semester Examinations, September/October - 2023

MATHEMATICS – IV

(Common to CE, EEE, CSE, IT)

Time: 3 Hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, Answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A

(25 Marks)

1.a) Define the differentiability of a complex function $f(z)$ at $z = z_0$. [2]

b) Define an entire function and give an example. [3]

c) State Cauchy's residue theorem. [2]

d) Define a pole of order m and give an example. [3]

e) Evaluate $\int_0^{2\pi} \frac{d\theta}{\sqrt{2}-\cos\theta}$ [2]

f) Define a bilinear transformation. [3]

g) Define a periodic function and give an example. [2]

h) State Fourier Integral theorem. [3]

i) Classify the equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + (1-\alpha)\frac{\partial^2 u}{\partial x \partial t}$ For different values of α. [2]

j) Solve by method of separation of variables $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ where $u(0, y) = 8e^{-3y}$. [3]

PART – B

(50 Marks)

2.a) Derive the C-R equations in polar form.

b) Is the function $u(x, y) = 2xy + 3xy^2 - 2y^3$ harmonic? [6+4]

OR

3.a) Determine the analytic function $f(z)$ such that $Re(f^1(z)) = 3x^2 - 4y - 3y^2$ and $f(1+i) = 0$.

b) Given that $u = \frac{\sin 2x}{\cosh 2y - \cos 2x}$. Find conjugate harmonic function of u. [5+5]

4.a) Determine $F(2), F(4)$, and $F(-3i)$ if $F(\alpha) = \oint_c \frac{5z^2-4z+3}{z-\alpha} dz$ where c is the ellipse $16x^2 + 9y^2 = 144$.

b) Find the Laurent's series expansion of $f(z) = \frac{\sinh 3z}{z^3}$ for $0 < |z| < \infty$. [6+4]

OR

5. Evaluate $I = \oint_C \frac{z^2+4}{z^3+2z.^2+2z}$ where C is (a) $|z| = 1$ (b) $|z + 1 - i| = 1$
(c) Rectangle with vertices $2 + i, 6 + i, 2 + 4i$, and $6 + 4i$. [10]

6. Evaluate $\int_{-\infty}^{\infty} \frac{cosmx}{e^x+e^{-x}}\, dx$. [10]

OR

7. Determine the linear fractional transformation that sends the points $z = 0, -i, 2i$ into the points $w = 5i, \infty, \frac{-i}{3}$ respectively. What are the invariant points of this transformation? Find the image of $|z| < 1$ (interior of a unit circle) under this transformation. [10]

8.a) Expand $f(x) = xsinx$, for $0 < x < 2\pi$, $f(x + 2\pi) = f(x) \forall x \in R$, as Fourier Series.

b) Find the Fourier sine transform of $e^{-|x|}$. [6+4]

OR

9.a) Obtain the half-range sine series for e^x *in* $0 < x < 1$.

b) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & \text{if } |x| < 1 \\ 0, & \text{if } |x| > 1 \end{cases}$. [5+5]

10. Solve the one dimensional wave equation by the method of separation of variables and write the favorable solution out of all the possible solutions. [10]

OR

11. An insulated rod of length l has its ends A and B maintained at 0^0C and 100^0C respectively until steady state conditions prevail. If B is suddenly reduced to 0^0C and maintained at 0^0C, find the temperature at a distance x from A at time t. [10]

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